Black holes and instabilities of negative tension branes

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We consider the collision in $2+1$ dimensions of a black hole and a negative tension brane on an orbifold. Because there is no gravitational radiation in $2+1$ dimensions, the horizon area shrinks when part of the brane falls through. This provides a potential violation of the generalized second law of thermodynamics. However, tracing the details of the dynamical evolution, one finds that it does not proceed from equilibrium configuration to equilibrium configuration. Instead, a catastrophic space-time singularity develops similar to the "big crunch" of $\Omega>1$ Friedmann-Robertson-Walker space-times. In the context of classical general relativity, our result demonstrates the instability of constructions with negative tension branes.

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I. INTRODUCTION

The idea that our three familiar spatial dimensions may exist as a submanifold of a higher dimensional space-time has opened a number of new avenues for string theory, particle physics, and cosmology [1–13]. Common to most of the recent incarnations is the condition that all interactions other than gravity be confined to our submanifold, or *brane*. Since departures from $(3+1)$-dimensional gravity are relatively difficult to constrain compared to those of other forces, this permits significant freedom to modify the gravitational interactions in the extra-dimensional space. It is in this modification of gravity and of the extra dimensions themselves that recent approaches differ from one another.

In many of the proposed models, the hierarchy problem is recast by bringing the fundamental mass scale of physics down to a weak scale. The large Planck mass observed on our 3 brane is then a derived quantity, the size of which arises from the relatively large volume of the extra-dimensional manifold. A striking consequence of this is that one expects quantum gravity or stringy effects to manifest themselves close to the electroweak scale. At first sight, this opens up many interesting possibilities for testing the idea of extra dimensions [14–24]. However, it is typically possible to construct these models consistently in such a way that they evade expected experimental tests.

One specific hurdle that extra-dimensional theories must clear is that the brane-bulk system should be a consistent, stable solution to Einstein gravity. In this paper we consider this issue for those constructions that include negative-tension branes. While the notion of negative energies is typically problematic, perturbative dynamical objections can be overcome by placing the offending brane at an orientifold plane.

In this paper we address two other issues associated with negative-tension branes, one of which is specific to those at orbifold fixed planes. The first has to do with their consequences for the generalized second law of thermodynamics, which states that the total entropy in matter and black holes does not decrease. In Einstein gravity this entropy can be written as

$$S_{\mathrm{TOT}} \equiv S_{\mathrm{matter}} + \frac{1}{4}A, \tag{1.1}$$

where A is the sum of the event horizon areas of all black holes, and we are using Planck units $(\hbar=c=k=G=1)$. In particular, any process which leaves S_{matter} fixed and decreases A leads to a violation of the generalized second law.

We begin with the simple observation that the area of a black hole event horizon increases when positive energy crosses the horizon, and decreases when negative energy crosses the horizon. A natural question then arises when one has a space-time with a negative tension brane: what happens if a black hole, initially far away from the brane, falls toward the brane and captures some of the brane within its horizon? One might expect that the generalized second law could be violated in this way, since the part of the brane that is swallowed by the black hole carries negative energy across the horizon.

In general, of course, whether the horizon actually shrinks will depend on what other matter or gravitational radiation is falling into the black hole. For this reason we consider a lower-dimensional system; a negative tension 1 brane in a $(2+1)$-dimensional space-time. The absence of gravitational radiation in $2+1$ dimensions will force the conclusion that the horizon shrinks when the black hole encounters a negative tension brane. This is so whether or not the brane sits at an orbifold.

This would provide a clear violation of the second law if the above collision connects two equilibrium configurations. Whether or not this is so turns out to be an interesting question that our low dimensional context allows us to explore in detail. In $2+1$ dimensions, the only black holes are the so called Bañados-Teitelboim-Zanelli [25] (BTZ) black holes, which arise only for negative cosmological constant. These black hole space-times can be constructed as a quotient of AdS space itself. Together with the lack of local gravitational degrees of freedom in $2+1$ dimensions, this fact allows us to construct a general solution describing the collision of a

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black hole and a brane. Interestingly, for branes at Z_2 orbifolds, the results depend dramatically on the sign of the brane tension.

We investigate a two-parameter family of solutions with negative tension branes in detail. This leads to our second issue. We find that the endpoint is not in fact an equilibrium configuration, but instead is a spacelike singularity similar to the ''big crunch'' of Friedmann-Robertson-Walker models with $\Omega > 1$ (and no cosmological constant). Continuity implies that a similar result occurs on an open subset of the full space of solutions, and it seems likely that the result is generic. This is naturally interpreted as a nonlinear dynamical instability of gravitating negative tension branes at orbifolds.

The paper is organized as follows. In Sec. II we provide a brief calculation of how the black hole horizon shrinks in our model. In Sec. III we then describe how to construct initial data for the brane–black hole system in AdS_{2+1}, and investigate the dynamics of the system in detail. This dynamics is qualitatively different in various regions of parameter space. We conclude in Sec. IV, and offer some interpretations of our results.

II. SHRINKING HORIZON

Our purpose in this section is to give a description of how a horizon evolves as it falls toward the negative tension brane. A black hole horizon is a closed null hypersurface. If we consider a congruence of null geodesics with tangent vector N^a orthogonal (and therefore tangent) to this hypersurface, then the divergence $\theta \equiv \nabla_a N^a$, describing the expansion or contraction of the congruence is described by the Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - R_{ab}N^a N^b, \qquad (2.1)$$

where σ_{ab} is the shear of the congruence, ω_{ab} is its twist, and λ is the affine parameter. (For a detailed description of this equation see, for example, Ref. [26].) The metric used to raise and lower the indices a and b is the induced metric on the null surface, which has a signature $(0,+,+)$. As a result, the twist term is positive definite. The shear term is related to the Weyl tensor, and hence to the existence of gravitational radiation—all of which vanish in $2+1$ dimensions. Therefore, from now on we set $\sigma_{ab} \equiv 0$.

Now an infinitesimal area element dA of the event horizon of the black hole is related to θ via

$$dA = dA_0 \exp\left(\int \theta(\lambda) d\lambda \right), \qquad (2.2)$$

and therefore the Raychaudhuri equation can be used to describe the evolution of the area as

$$\frac{d^2}{d\lambda^2}(\sqrt{A}) = \left(\omega_{ab}\omega^{ab} - \frac{1}{2}R_{ab}N^a N^b \right)\sqrt{A}. \qquad (2.3)$$

Using the Einstein equation, and recalling that N^a is null, we may rewrite this as

$$\frac{d^2}{d\lambda^2}(\sqrt{A}) = (\omega_{ab}\omega^{ab} - 4\pi T_{ab}N^a N^b)\sqrt{A}, \qquad (2.4)$$

where T_{ab} is the energy momentum tensor. Since the brane has negative tension, so long as no other matter is present we have

$$T_{ab}N^a N^b < 0 \qquad (2.5)$$

for null N^a not tangent to brane (i.e., the null energy condition is violated).

If the system were to reach equilibrium and a final horizon could be identified, the final expansion θ would vanish. It then follows from Eqs. (2.4) and (2.5) that θ would have been negative during the collision itself. In particular, no caustics can form on this surface during the collision, since immediately after a caustic the expansion of the corresponding null generator must be positive. Thus the null surface $\mathcal{S}^{\rm final}$ that eventually becomes the final horizon can be extended back to before the collision occurred, and we see that the area of $\mathcal{S}^{\rm final}$ was larger before the collision than it is afterward.

Similarly, suppose that the system began in equilibrium and that an initial horizon could be identified. Any outward directed null congruence outside this initial horizon has positive expansion at this stage, and we see that this will continue during the collision. Thus the null surface $\mathcal{S}^{\rm final}$ associated with the final horizon must begin inside the initial horizon. As a result, the initial area of $\mathcal{S}^{\rm final}$ must be smaller than that of the initial horizon. Since the area of $\mathcal{S}^{\rm final}$ decreases during the collision, the final area must be even smaller. We therefore arrive at a violation of the generalized second law if the system evolves from one equilibrium to another.

III. BRANE-BTZ BLACK HOLE SYSTEM

The discussion in Sec. II described the process of a black hole colliding with a negative tension brane, and traced the evolution of null congruences. On the surface, this would appear to yield a violation of the second law of thermodynamics. However, in order to identify particular null congruences as initial and final ''horizons,'' the evolution must connect two equilibrium configurations.

Whether or not this is the case in our $2+1$ setting is explored below through detailed investigation of the corresponding solutions. We require that our solutions contain a flat brane at an orbifold fixed plane and a BTZ black hole, but the space-time should be otherwise empty. Because $(2+1)$-dimensional gravity contains no local dynamics, the metric will be locally AdS everywhere except at the orbifold singularity.

At this point it is convenient to recall that the BTZ space-time is itself a quotient of AdS space. Similarly, recall that a space-time (the ''brane-only space-time'') describing the negative tension brane by itself consists of two copies of AdS patched together along the brane with Z_2 orbifold boundary conditions. Now of particular interest are cases in which the black hole and brane are initially separated. By

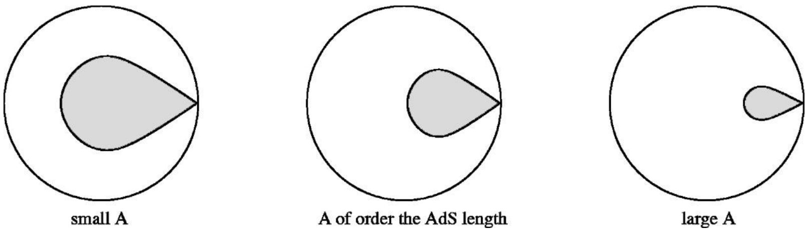

FIG. 1. The brane fundamental region at $t=0$ for three values of A.

this we mean that at least near some Cauchy surface $t=0$ there should be a region $\mathcal{R}_{\rm BTZ}$ of space-time enclosing our black hole in which the metric takes the standard BTZ form. Similarly, there should exist a region $\mathcal{R}_{\rm brane}$, near the brane. Since the space-time is to contain no further topological complications we may take the union $\mathcal{R}_{\rm brane}\cup\mathcal{R}_{\rm BTZ}$ to contain a Cauchy surface for the space-time and the intersection $\mathcal{R}_{\rm brane}\cap\mathcal{R}_{\rm BTZ}$ to be contractible.

Let us now count the number of parameters needed to describe our space-time. The tension of the brane is fixed by the cosmological constant, so there are no parameters associated with the brane region $\mathcal{R}_{\rm brane}$ itself. The BTZ region $\mathcal{R}_{\rm BTZ}$ is determined by two free parameters corresponding to the mass and spin of the BTZ black hole. Since the BTZ space-time and the brane-only space-time are by themselves locally AdS, the intersection $\mathcal{R}_{\rm brane}\cap\mathcal{R}_{\rm BTZ}$ is associated with an element of the AdS isometry group SO(2,2) which serves to patch the two regions together.

Now consider a ‘‘fundamental region’’ $\mathcal{F}_{\rm BTZ}$ of AdS space that covers the BTZ space-time exactly once under the BTZ quotient map. Further, in the brane-only space-time the AdS region on one side of the brane is related to that on the other side through the orbifolding procedure. Hence we refer to the region on one side of the brane as the ‘‘fundamental region’’ $\mathcal{F}_{\rm brane}\subset$AdS. We may therefore write $\mathcal{R}_{\rm BTZ}\subset\mathcal{F}_{\rm BTZ}$ and $\mathcal{R}_{\rm brane}\subset\mathcal{F}_{\rm brane}$, provided that we keep in mind the appropriate identifications.

The choice of $\mathcal{F}_{\rm BTZ}$ and $\mathcal{F}_{\rm brane}$ is, of course, unique only up to an element of the AdS symmetry group SO(2,2). However, we may use this symmetry to bring one of the fundamental regions into a standard configuration. Thus, only that element of SO(2,2) describing the relative configuration of $\mathcal{F}_{\rm BTZ}$ and $\mathcal{F}_{\rm brane}$ in AdS is of interest. This is precisely the freedom associated with the overlap of $\mathcal{R}_{\rm BTZ}$ and $\mathcal{R}_{\rm brane}$ in our space-time.

Consider, then, the set of all space-times constructed by specifying BTZ and brane fundamental regions $\mathcal{F}_{\rm BTZ}$ and $\mathcal{F}_{\rm brane}$ in AdS_{2+1}, taking the intersection $\mathcal{F}_{\rm BTZ}\cap\mathcal{F}_{\rm brane}$ and making appropriate identifications. We have shown that all of the space-times we seek will lie in this set. However, some space-times constructed in this way may fail to be smooth or may fail to contain separated branes and black holes. Thus we may say that the identification of $\mathcal{F}_{\rm BTZ}$ and $\mathcal{F}_{\rm brane}$ is necessary but not sufficient for building the space-times of interest.

It will be enough for us to analyze the most straightforward case in which $t=0$ is a moment of time symmetry. From now on we will assume that this condition holds. As a result, we now restrict consideration to spinless BTZ black holes.

A. Initial data

Much of the physics of this problem can be understood by studying the initial data associated with the moment of time symmetry. If t is a global time coordinate on AdS_{2+1}, we may take this moment to occur in the slice $t=0$. Here it is convenient to consider AdS_{2+1} as a solid cylinder, so that the $t=0$ surface is a disk. Given the condition that $\mathcal{F}_{\rm BTZ}$ and $\mathcal{F}_{\rm brane}$ respect this time symmetry, the identifications of AdS used to make the BTZ or brane space-times preserve the surface $t=0$. As a result, we may speak about the fundamental domains $\mathcal{F}^0_{\rm BTZ}$ and $\mathcal{F}^0_{\rm brane}$ associated with the corresponding quotient of the $t=0$ slice.

The space of time symmetric initial data is labeled by four parameters. To see this, recall that the black hole spin has been set to zero and that the requirement that the brane and black hole both be present at $t=0$ forces the SO(2,2) parameter used to patch together $\mathcal{R}_{\rm BTZ}$ and $\mathcal{R}_{\rm brane}$ to take values in SO(2,1), the symmetry group of the $t=0$ surface itself. However, it is sufficient for our purposes to work with the two parameter subfamily in which we require the space-time to have an additional Z_2 symmetry as described below. This requirement breaks the SO(2,1) down to just the one-dimensional subgroup associated with one of the boost generators, and we may think of the associated free parameter as the location of the brane in Poincaré coordinates.

For definiteness we will work in global coordinates (t,ρ,ϕ) in which the AdS line element takes the form

$$ds^2=\frac{4l^4}{(l^2-\rho^2)^2}\left[-\left(\frac{l^2+\rho^2}{2l^2}\right)^2dt^2+d\rho^2+\rho^2d\phi^2\right], \tag{3.1}$$

with $t\in[-\pi/2,\pi/2]$, $\rho\in[0,1]$ and $\phi\in[-\pi,\pi]$. These are the coordinates referred to as ‘‘sausage coordinates’’ in Ref. [27], which is a useful reference for visualizing the BTZ fundamental region. The extra Z_2 symmetry we impose corresponds to invariance under $\phi\rightarrow-\phi$.

Using these coordinates, $\mathcal{F}^0_{\rm brane}$ and $\mathcal{F}^0_{\rm BTZ}$ are drawn in Figs. 1 and 2, respectively. The fundamental domains are the unshaded regions. Each figure shows three sample cases illustrating what happens when we change the free parameter associated with the respective fundamental region. In Fig. 1, the parameter (which we call A) may be thought of as the location of the brane in Poincaré coordinates. The three cases shown there are of course related by the action of an AdS isometry, but this will shortly be broken by the presence of the black hole. In Fig. 2, the parameter is the mass M of the BTZ black hole, which controls the ‘‘size’’ of the fundamental region as shown. The dotted line in Fig. 2 is the intersec-

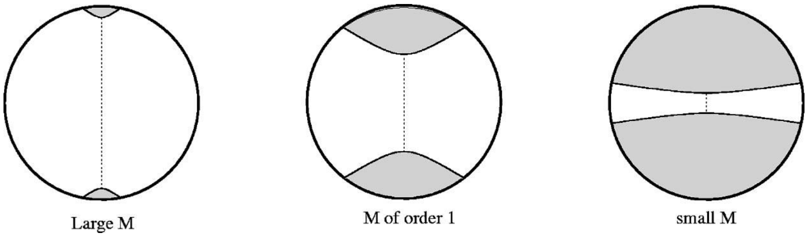

FIG. 2. The BTZ fundamental region at $t=0$ for three values of M.

tion of the black hole horizon with the $t=0$ surface. This intersection is the bifurcation surface of the horizon.

It remains to put these domains together in a consistent way. We are interested in configurations in which the black hole and brane are separated at $t=0$, meaning that the brane lies entirely on one side of the black hole horizon. It is clear that for any black hole mass M one can choose a large enough value of A, so that the objects are indeed separated. Note that rotating the disks in Fig. 2 by the action $\phi \rightarrow \phi + \pi/2$ would also yield a configuration compatible with our symmetries, but that it would then be impossible to separate the brane and the black hole.[1]

It is interesting to note that positive tension branes behave very differently. We may represent a positive tension brane as in Fig. 1 above, but with the shaded region now representing the fundamental region. A glance at Figs. 1 and 2 shows that it is now impossible to separate brane and black hole. This is clearly a reflection of the well-known statement that there is only a ''small'' region of AdS that is far from a positive tension brane on a Z_2 orbifold.

B. Dynamics and collisions

In Sec. III A we established the existence of initial data representing a separated, momentarily static configuration containing a black hole and a negative tension brane. We now wish to study the time evolution of this solution. Certain qualitative features are readily deduced from the well-known properties of black holes and branes with respect to the global coordinates (t,ρ,ϕ) on AdS_{2+1}. For example, as t increases the black hole horizon expands outward to both right and left, while the boundaries of the BTZ fundamental region move toward each other (see, e.g., Ref. [27]). On the other hand, the brane expands, with the ends separating and moving along the boundary at the speed of light. A typical configuration of both black hole and brane for $t>0$ is shown in Fig. 3.

The expansion of the brane and the contraction of the BTZ fundamental region clearly results in contact between the boundaries of $\mathcal{F}_{\rm BTZ}$ and $\mathcal{F}_{\rm brane}$. The points of contact will be conical singularities in the resulting space-time. If we simply extrapolate the behavior of these boundaries it would appear that the entire space-time outside the black hole would quickly disappear into such a singularity. However, once the singularity forms it is no longer clear that we can follow the evolution using classical general relativity. It is therefore important to discover whether events along this potential singularity are causally connected, so that we may determine whether modifications to general relativity at one such point could affect the formation of the rest of the singularity.

This turns out to be simplest to analyze in Poincaré coordinates, which we label γ, β, and z. The relevant metric is most easily obtained by making use of ''embedding'' coordinates as an intermediate step. Recall that AdS_{2+1} may be described as the surface

$$T^2+U^2-X^2-Y^2=l^2, \tag{3.2}$$

embedded in $(2+2)$-dimensional Minkowski space, with line element

$$ds^2=-dT^2-dU^2+dX^2+dY^2. \tag{3.3}$$

In Eq. (3.2), l is the AdS length scale. The embedding coordinates (T,U,X,Y) are related to our global coordinates via

$$\begin{aligned} X&=-\frac{2l^2\rho}{l^2-\rho^2}\cos(\phi), \quad T=l\left[\frac{l^2+\rho^2}{l^2-\rho^2}\right]\sin\left(\frac{t}{l}\right),\\ Y&=\frac{2l^2\rho}{l^2-\rho^2}\sin(\phi), \quad U=l\left[\frac{l^2+\rho^2}{l^2-\rho^2}\right]\cos\left(\frac{t}{l}\right). \end{aligned} \tag{3.4}$$

The Poincaré coordinates can then be expressed as

$$z=\frac{1}{U+X}, \quad \beta=\frac{Y}{U+X}, \quad \gamma=\frac{T}{U+X}, \tag{3.5}$$

with the resulting line element

$$ds^2=\frac{1}{z^2}(-d\gamma^2+d\beta^2+dz^2). \tag{3.6}$$

These coordinates have two particularly useful simplifying properties. First, the metric [Eq. (3.6)] is conformal to the

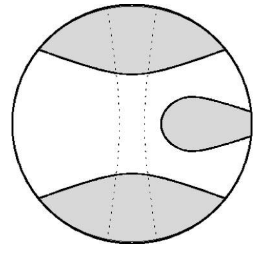

FIG. 3. A typical configuration of $\mathcal{F}_{\rm BTZ}$ and $\mathcal{F}_{\rm brane}$ at $t>0$.

[1]One can also show that such configurations are singular at $t=0$.

Minkowski metric and, second, these coordinates may be chosen so that the brane is located on the hypersurface $z=A$.

Note that in our various coordinate systems the moment of time symmetry can be equivalently characterized as $t=0$, $T=0$, or $\gamma=0$. In addition, the Z_2 symmetry $\phi\rightarrow-\phi$ can be represented in Poincaré coordinates as $\beta\rightarrow-\beta$.

We must now identify the boundary of the BTZ fundamental region in Poincaré coordinates. To describe $\mathcal{F}_{\rm BTZ}$, it is useful to introduce the parameter

$$\alpha\equiv\frac{1}{l}\tanh(\pi\sqrt{M}). \tag{3.7}$$

It is easily seen [27] that a useful choice for the boundary of the BTZ fundamental region consists, in embedding coordinates, of the two surfaces

$$Y=\pm l\alpha U. \tag{3.8}$$

Translating this into Poincaré coordinates we obtain

$$\pm\beta=\frac{1}{2}\alpha(l^2+z^2+\beta^2-\gamma^2), \tag{3.9}$$

with the positive sign describing the upper boundary and the negative sign the lower one. From now on, we shall follow only the upper boundary, as the two are related by our Z_2 symmetry.

Recall that AdS_{2+1} in Poincaré coordinates is conformally equivalent to Minkowski space with metric $-d\gamma^2+d\beta^2+dz^2$. The singularity forms on the line where the brane ($z=A$) intersects the surface described by Eq. (3.9). In terms of the conformally rescaled space, it is clear that this singularity propagates in a straight line (along $z=A$) at a speed determined by Eq. (3.9) with $z=A$. A short calculation shows that this speed is

$$\frac{d\beta}{d\gamma}=\frac{\alpha}{\alpha\beta+1}\sqrt{l^2+A^2+\left(\frac{\alpha\beta+1}{\alpha}\right)^2-\frac{1}{\alpha^2}}. \tag{3.10}$$

Further, the condition that the brane and the boundary of $\mathcal{F}_{\rm BTZ}$ be separated at the moment of time symmetry ($\gamma=0$) yields

$$l^2+A^2-\frac{1}{\alpha^2}>0, \tag{3.11}$$

which, when substituted into Eq. (3.10), yields $d\beta/d\gamma>1$. Consequently, the singularity propagates in a straight line at a velocity greater than that of light in the conformally rescaled metric. In fact, the singularity lies on a surface that, in Poincaré coordinates lies in the $z=A$ plane on the two hyperbolae[2] of constant proper time $\sqrt{l^2+A^2-(1/\alpha^2)}$ from the events $z=A$, $\beta=\pm1/\alpha$, and $\gamma=0$. It follows that no two points on the singularity are in causal contact.

To better understand this, note that Figs. 1 and 2 show that, for any brane position A, one can in fact choose a black hole small enough that the boundaries of the two fundamental regions contact at $t=0$, or as close to $t=0$ as one would like. In particular, this occurs "first" outside the horizon. One can also show that for $A^2>\alpha^{-2}(\alpha^{-2}-1)$ the singularity first develops inside the horizon if time ordering is defined using the Poincaré coordinate γ. However, since no two points along the singularity are in causal contact, we see that this ordering is of little relevance. Therefore, the singularity is best thought of as arising "simultaneously" throughout the space-time.

Our conclusion must hold in the original AdS metric as well. It follows that the collision of the black hole and brane ends not in a black hole attached to the brane, but instead in a spacelike singularity in which much of the universe is destroyed. The attentive reader will have noticed something interesting about the line $\beta=0$, $z=A$, which may be thought of as the "leading piece of the brane" in the global coordinates of Fig. 3. This line reaches the singularity at the finite time:

$$\gamma_{\rm leading}=\sqrt{1+A^2}. \tag{3.12}$$

It may seem odd that this event is not on the edge of the AdS space. However, one can readily show that the event does lie on the singularity of the BTZ black hole, in particular between the two horizons. Thus we see that the collision-induced singularity engulfs the entire exterior space-time on the side of the black hole that contains the brane, and that this singularity joins on to the black hole singularity. In fact, one could perhaps interpret this result as extending the black hole singularity out to the edge of the space-time, destroying the future asymptotic region and making the concept of an event horizon meaningless on the right side of the black hole.

In any case, one sees that our singularity does not extend into the left exterior region of the BTZ space-time. The left exterior is completely unaffected by the presence of the brane in the right exterior. This is as it should be, since the black hole horizons should prevent any influence from propagating from the right exterior to the left one. The appearance of this singularity seems to demonstrate an instability of the system that is perhaps of even more interest than the potential failure of thermodynamics described in Sec. II.

IV. DISCUSSION AND CONCLUSIONS

We have illustrated two difficulties that arise when negative tension branes encounter black holes. The first is a potential violation of the generalized second law of thermodynamics that can occur whether or not the brane is located at an orbifold fixed plane. Note that if the negative energy stored in the brane tension could be transformed into some other sort of energy, then one could also use it to violate the second law in systems without black holes. For example, if the negative energy could be used to cool a hot star, this cooling would seem to lead to a decrease in entropy. The

[2]These hyperbolae intersect at $\beta=0$. To form the singular surface, one should include only those pieces of each hyperbola which do not lie to the future of any event on the other hyperbola.

important feature of black holes is thus that any form of energy that enters them directly affects the horizon area (and thus the entropy). The second difficulty we investigated is a nonlinear dynamical instability that causes the entire space-time outside the black hole to collapse when the brane is located at an orbifold fixed plane.

We have seen that horizons of $(2+1)$-dimensional black holes shrink when encountering a negative tension brane if no other matter is present. This follows from the identical vanishing of the shear of a null congruence in $2+1$ dimensions. One expects similar behavior to occur in higher dimensions, unless for some reason a significant quantity of gravitational radiation is also present so that the shear term becomes important in the Raychaudhuri equation.

This results in a clear violation of the second law of thermodynamics if the collision connects two equilibrium configurations. Ideally, we would like to begin with a black hole well separated from the brane and end with some well-defined configuration. In Sec. III we found that $2+1$ space-times containing a moment of time symmetry with well-separated BTZ black holes and negative tension branes do in fact exist. The black hole and brane then begin to fall toward one another. The result of this collision is the collapse of the entire space-time outside the black hole in a spacelike singularity. Although we have explicitly discussed only a two-parameter subfamily of the full four-parameter family of such time symmetric initial data, continuity tells us that the same result follows for an open subset of such data containing our subfamily. Similarly, the conclusion must hold for an open subset of the full space of initial data, which would allow for spinning black holes as well.

Let us note that the singularity forms along the brane, and let us recall that it can be thought of as forming simultaneously across the entire space-time outside the black hole. As a result, it is clear that at least some of the brane falls through the horizon of the black hole before the singularity develops. The black hole then proceeds to ''eat'' the brane and reduce in size. Where the horizon intersects the final singularity, it is of zero size. The area of the piece of brane that falls through the horizon before the singularity occurs can be calculated. Multiplying by the brane tension gives

$$\mu_{\rm brane} = -2\frac{A}{l}\sqrt{\frac{1}{\alpha^2} - l^2}, \tag{4.1}$$

which, roughly speaking, is the amount of negative brane mass that has fallen though the horizon. One can show that this is much greater than M. It is natural to associate this excess with the kinetic energy that the system acquires during the time in which the brane and black hole are falling toward each other so that the sum of M, the kinetic energy, and $\mu_{\rm brane}$ vanishes.

We would like to interpret the singularity formed in the collision as describing a nonlinear instability in the presence of negative tension branes on Z_2 orbifolds. However, this raises certain questions. First, we have seen that, for a fixed ''brane location'' A, separated black holes arise only when the mass of the black hole is sufficiently large. As a result, one might be tempted to think that any such instability is triggered only by large disturbances. Recall, however, that the brane location A is not physically meaningful in and of itself. There is in fact an AdS isometry that simply scales the Poincaré coordinates, taking our brane to an arbitrary new value of z. Thus, the correct statement is that black holes of any size are allowed so long as they are sufficiently far away from the brane.

In addition, it is known that BTZ black holes can form dynamically from the collision of matter [28]. One would therefore naively expect that arbitrarily small BTZ black holes could in fact be formed even close to negative tension branes. The natural conclusion is that the formation process can begin near the brane, but that a singularity of the sort discussed here arises before this process is complete. Thus we expect nonlinear instabilities with negative tension branes in the presence of matter fields even if no black holes are initially present.

One may ask whether the same sort of singularity that we found in $2+1$ dimensions also occurs in higher-dimensional cases. After all, we have shown that no smooth static solutions exist for black holes on positive tension branes in the $(2+1)$-dimensional case, while such solutions have constructed in $3+1$ dimensions [29].

Let us first recall that certain black holes [30–32] in higher dimensions are also asymptotically AdS, and can be represented as quotients of AdS space analogous to those that yield the BTZ black hole in $2+1$ dimensions. Collisions of such black holes with negative tension branes are quite similar. The singularity again lies in the $z=A$ plane on hyperbola of constant proper time $\sqrt{l^2+A^2-(1/\alpha^2)}$ from the events at $z=A$ and $\beta=\pm 1/\alpha$, and so forms a causally disconnected set.[3] However, black holes constructed in this way have an unusual topology for both the horizon and any associated anti–de Sitter region [33–35]. As a result, such black holes do not form from the collapse of matter subject to normal boundary conditions. These examples are therefore not sufficient to argue convincingly for a dynamical instability in higher dimensions. Clearly, a productive line of investigation in $3+1$ dimensions would be to perform a stability analysis of the solution found in Ref. [29], describing a black hole on a negative tension $2+1$ brane.[4]

Finally, let us recall that negative tension objects are often motivated by considering orientifold constructions of string theory. Although orientifolds do indeed have negative ten-

[3]One difference, however, follows from the fact that the unusual horizon topology means that these black holes have only a single exterior region. In some sense, the two exteriors of the $(2+1)$-dimensional BTZ hole become connected. As a result, the fact that the left BTZ exterior was not affected by the brane translates into the statement that the collision-induced singularity in the higher-dimensional analogues does not engulf the entire exterior region. However, it does engulf the entire brane.

[4]Comparing the entropies of black holes on and off negative tension branes (as done in Ref. [29] for positive tension branes) does suggest that the black hole has less entropy on the brane. This is in turn suggestive of a dynamical instability. We thank Roberto Emparan for this comment.

sion, it is important to note two further properties. First, when orientifolds arise in a space-time that is locally asymptotically flat, the orientifolds have enough supersymmetry (see, e.g., Ref. [36]) to allow one to show that they represent a state of minimal energy for their charge.[5] As a result, one strongly expects that they are dynamically stable even in the presence of (at least uncharged) black holes. Second, let us consider a particularly well understood orientifold known as the O6 plane. When the coupling is strong enough, this orientifold corresponds to a smooth classical 11-dimensional space-time [37–39] which is $\mathbf{R}^7$ times the Atiyah-Hitchin metric [40]. This space-time is Ricci flat, and satisfies the vacuum Einstein equations. As a result, one may describe the collision of a black hole with such an orientifold as a problem in pure Einstein-Hilbert gravity. Under such conditions, the Raychaudhuri equation leads in the usual way [41] to the conclusion that the total horizon area increases. As a result, our violations of the generalized second law of thermodynamics will not arise in this context. As the various stringy negative tension orientifolds are related by T duality, one expects that the other orientifolds of string theory also have properties such that the second law of thermodynamics is upheld in collisions with black holes. Since we have seen that the orbifold boundary condition itself is not sufficient, it would be interesting to understand in more detail just what properties of these orientifolds enforce the second law. For example, it may be due the special form of the coupling of an orientifold to the dilaton or gauge fields of string theory.

In summary, we have seen that the orbifold boundary condition of, e.g., Ref. [7] is not sufficient by itself to render negative tension branes stable. Our results indicate that both thermodynamic and nonlinear dynamical instabilities remain. As a result, if use is to be made of negative tension branes in various models, it is important first to show that the particular branes being used are immune from these effects. A conservative working hypothesis might be that only the rather specific negative tension branes that arise as supersymmetric orientifolds of string theory should be considered.

[5]It is in fact plausible that orientifolds represent the absolute lowest energy state compatible with the asymptotic metric.

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